Problem III.1 ... it's too dry in here

3 points; průměr 2,61; řešilo 147 studentů

Danka has a humidifier in her dorm room, which evaporates water from its boiling point to create warm steam. The device can hold a maximum of $V = 3.8\,\mathrm{l}$ of water, which it uses up in $t = 24\,\mathrm{h}$. What is its efficiency, i.e., what fraction of the energy drawn from the electrical grid it uses to convert the water to steam? The input power of the humidifier is $P = 260\,\mathrm{W}$, and Danka put water at $T_0 = 20\,^\circ\mathrm{C}$ inside. All the necessary properties of water can be looked up. *Danka has to use a humidifier in her dorm room during winter.*

To find out the efficiency of the humidifier, we first need to find out how much energy is necessary to evaporate the water in the humidifier. This energy is the sum of the heat used to heat the water to the boiling point, i.e., temperature $T_1 = 100\,^\circ\mathrm{C}$, let us denote it by Q, and the heat of vaporization L. According to known formulae, we can write

$$Q = V\rho c(T_1 - T_0)\,,$$

and

$$L = V\rho l_\mathrm{v}\,.$$

In the tables we can find the specific heat capacity of water $c = 4\,182\,\mathrm{J{\cdot}kg^{-1}{\cdot}K^{-1}}$ and its heat of vaporization $l_v = 2\,257\,\mathrm{kJ{\cdot}kg^{-1}}$. We consider the density of water to be $1\,000\,\mathrm{kg{\cdot}m^{-3}}$. Now, we just need to calculate the total electrical energy that the humidifier consumes in 24 hours. We calculate it as

$$E = Pt\,.$$

For the efficiency of the humidifier, we get

$$\eta = \frac{Q + L}{E}\,,$$
$$\eta = \frac{V\rho[c(T_1 - T_0) + l_\mathrm{v}]}{Pt}\,.$$

After converting the values to the correct units and substituting them into the equation, we finally have

$$\eta \doteq 43.8\,\%\,.$$

Daniela Dupkalová
daniela@fykos.org

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