Problem II.2 ... inflated tire

3 points; (chybí statistiky)

It is said that if you want to inflate a car's tires, you should do it when they are cold. Therefore, Jarda drove to a petrol station with a compressor, bought a hot dog, and waited for the tires to cool down. Curious, he measured the tire pressure before and after his snack. It had dropped from 2.7 bar *to* 2.5 bar*. He wondered whether the tire pressure could be determined by the height of the car's body above the road. How much did the body of Jarda's car approach the ground due to the decrease in the tire temperature? The weight of the car is* 1.3 t*. The outer radius of the tires is* 32 cm*, the inner radius is* 22 cm*, and their width is* 21 cm*. Assume that the tires deform due to the car's weight only on the underside where they touch the ground.*

Jarda would drain his soul (as well as his bicycle's inner tube) for FYKOS.

In the area where the tires of the car are in contact with the ground, weight and pressure forces are equal, so

$$mg = 4Sp\,,$$

where m is the mass of the car, p is the pressure inside the tires, and S is the area of contact. According to the problem statement, we assume the tire has a circular shape cut off at the bottom. We can calculate the length of this chord from the area S. Then, using the Pythagorean theorem, we find its distance from the center of the tire

$$x = \sqrt{R^2 - \left(\frac{S}{2h}\right)^2}\,,$$

where R is the outer radius of the tire, and h is its width.

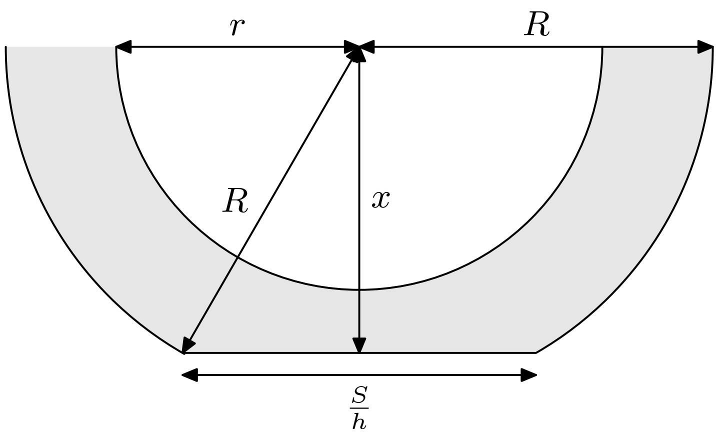

Fig. 1: Sketch for the calculation of the distance x of the ground from the centre of the tire.

For warmer and colder tires, we can find the difference between the distances x as

$$\Delta x = \sqrt{R^2 - \left(\frac{mg}{8hp_1}\right)^2} - \sqrt{R^2 - \left(\frac{mg}{8hp_2}\right)^2}\,.$$

We get the correct result by substituting into this expression. However, since $R^2 \gg (mg/(8hp_i))^2$, where $i \in \{1, 2\}$, we can use a mathematical tool called Taylor series. It says that for $x \ll 1$ we can, with good precision, write

$$\sqrt{1+x} \approx 1 + \frac{x}{2}\,.$$

To obtain R^2, we first extract it from the square roots and then proceed with the approximation process mentioned earlier. After several straightforward steps, we get

$$\Delta x \approx \frac{1}{2R}\left(\frac{mg}{8h}\right)^2\left(\frac{1}{p_2^2} - \frac{1}{p_1^2}\right) \doteq 0.2\,\mathrm{mm}\,.$$

Numerically, we got the same result as if we had plugged the values into the precise equation for Δx. The error, however, is far less than the accuracy of the input values, and the plugging in is simpler. The Taylor expansion is often used in physics in these situations precisely because of the noticeable simplification of expressions.

We can, therefore, state that under normal conditions, it is impossible to determine the inflation of a tire from the height of a car above the road since the change is minimal.

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