Problem I.4 ... truck flip

7 points; průměr 3,85; řešilo 62 studentů

Legolas had a dream in which the truck braked so quickly that the container lifted off the ground and did a somersault over the cab. He wondered if that was possible, so he tried to do the math. In his model, the entire truck has a mass of m and comprises a tractor and a container. It can rotate freely in all directions around the point where it is connected to the tractor. When the truck is on a flat road, the center of gravity of the container is h above this connection and at a distance l from it. Depending on the slope of the road φ, how much force must the truck brake in order to lift the wheels under the container off the road? *Lego was literally dreaming*

In this case, the axis of rotation goes through the point in which the container is connected to the tractor. Therefore, to determine when the rear wheels lift off the road, we need to determine the total torque acting on the container relative to this point in the system of reference decelerating with the truck.

In this system, two forces are acting on the truck – the weight and the inertial force. Both of those forces can be transferred to the center of gravity. Let us denote the mass of the container m_k; the weight then has a magnitude of $m_\mathrm{k}g$ and acts vertically downwards. If the road has a slope of φ, the weight component parallel to the path is $m_\mathrm{k}g\sin\varphi$, and the perpendicular one is $m_\mathrm{k}g\cos\varphi$. The moment arm of the component in the direction of the road has a length of h, and the moment arm of the component perpendicular to the road has a length of l.

It is also very important to think about the signs of the terms. If the truck rides downhill, the torque of the component parallel to the road is acting against the torque of the perpendicular component (as can be seen from the fact that, theoretically, for a very steep slope, the truck could overturn even if it were stationary…). Let us say that positive φ corresponds to the case where the truck goes downhill, then the torque of the weight will be $M_\mathrm{g} = m_\mathrm{k}g(l\cos\varphi - h\sin\varphi)$. We see that the torques add up for negative φ (that is, the case of the truck going uphill), which fits our expectations.

The inertial force in a non-inertial system decelerating with the truck with an acceleration a has a magnitude of $m_\mathrm{k}a$ and acts in the direction of the truck movement; therefore, its torque has a magnitude of $M_a = m_\mathrm{k}a\,h$ and (for “reasonable” slopes of the road) is acting against M_g.

Let us denote the resulting torque $M = M_\mathrm{g} - M_a$. When the torque is positive, the rear wheels need to compensate for it by pushing on the road (that is what they are there for, after all). If the torque is negative, the wheels are not stuck to the road, so they will lift off. The borderline case occurs when the torque is zero – at that moment, the rear wheels no longer press against the roads, and for any more braking, the wheels lift off the road.

Now, let us calculate for which acceleration a is M zero

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\begin{aligned}
0 = M = M_\mathrm{g} - M_a &= m_\mathrm{k}g(l\cos\varphi - h\sin\varphi) - m_\mathrm{k}a\,h\,,\\
ah &= g(l\cos\varphi - h\sin\varphi)\,,\\
a &= g\frac{l\cos\varphi - h\sin\varphi}{h} = g\left(\frac{l}{h}\cos\varphi - \sin\varphi\right)\,.
\end{aligned}
$$

One might think that the only thing to do now is to substitute $F = ma$, but that is not true! It is valid only for $\varphi = 0$ because, otherwise, the truck will tend to accelerate down the slope, and we want to know with what force it must brake. If it goes downhill, it must brake to compensate for acceleration the truck would have had under the hill without braking, and it has to brake with the acceleration a on top of that. Conversely, when the truck goes uphill, it needs to brake less since the weight slows it down. Let us denote the acceleration with which the truck would ride without braking by a_0, then the force with which it must brake is $F = m(a + a_0)$, where a_0 is positive for φ positive and vice versa.

The whole truck has a mass of m, so the component of weight in the direction of the road in $F_\mathrm{k} = mg\sin\varphi$ if it would not brake, its acceleration would be $a_0 = F_\mathrm{k}/m = g\sin\varphi$. Therefore, the force with which the truck must brake to lift its rear wheels off the road must be greater than

$$F = m(a + a_0) = m\left(g\left(\frac{l}{h}\cos\varphi - \sin\varphi\right) + g\sin\varphi\right) = mg\frac{l}{h}\cos\varphi\,.$$

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