## Problem I.3 ... new bicycle

5 points; průměr 3,29; řešilo 189 studentů

*A cyclist with the mass $m_\mathrm{c} = 62.3\,\mathrm{kg}$ started riding his bike at constant power from rest to the wanted speed at time $t = 103\,\mathrm{s}$. His bicycle's steel frame and fork have a mass $M = 6.50\,\mathrm{kg}$, and each of the two wheels has a mass $m = 1\,950\,\mathrm{g}$. How long would it take him to get going on a bike with a carbon frame and fork that is four times lighter? The weight of the other bicycle parts is included in the cyclist's weight.* *Dodo borrowed his sister's bike.*

When starting up, the cyclist must supply the bike with enough energy to put the bike into translational motion and spin the wheels. For the kinetic energy of the translational motion, we can express as

$$E_1 = \frac{1}{2}m_\mathrm{c}v^2 + \frac{1}{2}Mv^2 + \frac{1}{2}2mv^2\,,$$

where $m_\mathrm{c}$ is the mass of the cyclist, $M$ is the mass of the frame, and $m$ is the mass of each wheel.

A part of the energy is in the form of the rotational energy in the wheels. If we assume that all of the mass of the wheel is at its edge at a distance $R$ from the axis of rotation, we get the energy contribution of the rotational motion

$$E_2 = 2\frac{1}{2}mR^2\omega^2 = mv^2\,,$$

where we used the relation between angular and circumferential velocity $v = R\omega$ valid for a non-slipping wheel. The total energy is

$$E = E_1 + E_2 = \frac{1}{2}m_\mathrm{c}v^2 + \frac{1}{2}Mv^2 + 2mv^2\,,$$

which we deliver in time $t$ with a constant power of size $P = E/t$. In the second case, we can express the time as

$$t' = \frac{E'}{P} = t\frac{E'}{E} = t\,\frac{\frac{1}{2}m_\mathrm{c} + \frac{1}{2}M' + 2m}{\frac{1}{2}m_\mathrm{c} + \frac{1}{2}M + 2m}\,,$$

where $M' = M/4$ is the new mass of the bicycle frame. After substantiating the numerical values, we get $t' = 96.4\,\mathrm{s}$. We can see that the cyclist will start faster on the lighter bicycle, but the change is not very large. We have only saved about seven seconds by lightening the bike by almost five kilograms. From the final relation, we can see that it is better to save on the mass of the wheels, which, compared to the non-rotating parts of the bicycle, contribute twice as much.

***Jozef Lipták***
liptak.j@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.