## Problem VI.P ... Earth at full throttle

10 points; průměr 6,48; řešilo 31 studentů

*Estimate the upper limit of work that can be done on Earth over the long term. The planet must remain habitable and, if possible, with the same climate for future generations.*

*Jáchym's laptop is overheating.*

### *Introductory words*

For starters, we must think carefully about the interpretation of the assignment, because how we approach the problem and what factors will limit us will depend on that. Energy will be the main talking point in the text. Electrical energy will be the most important for us, as it is easily convertible into other forms of energy or work.

**Ecology, biosphere, and climate** While the planet is to remain habitable, our work and acquisition of energy cannot seriously disrupt Earth's biosphere. From this point of view, it is inappropriate, for example, to cover the entire surface with solar panels because they would absorb all the incoming solar radiation, and there would be none left for the plants. Surely, we do not need to elaborate further that without plants, the entire food chain would subsequently collapse, most life forms would gradually die out, and the planet would become uninhabitable. Therefore, we will need high-power energy sources that do not take up much space, or that we can place under Earth's surface.

The average temperature of the planet is the second important factor for the biosphere. Several factors influence temperature. One of the most important is global warming, which is related to the amount of greenhouse gases in the atmosphere. If we are to produce energy and do work in the long term (we will talk about how much that is later), then we necessarily want to emit as few greenhouse gases as possible, preferably none. Fossil fuels are unsuitable from this perspective (they are also inappropriate from a sustainability perspective, but more on that later).

Most experts agree that great economic, social and ecological damage will occur if the Earth's average temperature rises by 2 °C in a short period of time.[1]

**Sustainability** We require a reliable and sustainable energy source to carry out our work in the long term. However, what do we mean by sustainable? We may have imagined a renewable energy source, and this is understandable as it has been instilled in us since forever. The truth is that there is no such thing as a renewable energy source. Energy cannot be produced; it can only be converted from one form to another that we can use more efficiently. Wind power plants harness the energy of the wind or air currents, while hydroelectric power plants use the energy of the water cycle. For instance, wind is generated by pressure differences, and these can be caused by variations in temperature. Solar energy can provide the temperature difference. Evaporation is a significant part of the water cycle, and the Sun supplies the energy for it. However, even the Sun will eventually stop supplying energy because, in approximately 5 billion years, it will consume most of its fuel (hydrogen and helium) and become a white dwarf. Hence, even stars cannot be considered a renewable energy source.

The Second Law of Thermodynamics also plays a role here, which implies, among other things, that the whole universe is heading towards an inevitable "heat death". We shall come back to this law.

[1] `https://www.britannica.com/science/global-warming`

However, we can talk about stars and the aforementioned *renewable* energy sources in the context of sustainable energy because we can harness them for orders of magnitude more than tens of thousands of years. Let's say that we would consider an energy source sustainable if it lasted for more than a thousand years (during which time technology would have advanced anyway, and physicists would look at this task differently).

**Energy source** We will need a powerful energy source that does not take up much space. Renewable energy sources are unsuitable, due to their unreliability (production depends on weather) and the need for a large amount of space (low power density). A more suitable candidate seems to be fission or fusion nuclear power.

### *Upper estimate model*

In the next section, we propose a model that attempts to estimate an upper bound on the possible doing of work, regardless of whether we can obtain the necessary energy. However, we need to consider some constraints on the energy acquisition process, especially the Second Law of Thermodynamics. Our model mainly focuses on the amount of waste heat energy that the Earth can radiate to the surroundings without the average temperature at its surface rising by 2 °C.

**Earth's current energy balance - without doing any work** The average temperature of the Earth's surface in the pre-industrial era was 13.7 °C.[2] At the same time, we know that there is $P_\mathrm{S} = 173\,000\,\mathrm{TW}$ of energy from the Sun hitting the Earth, and another $P_\mathrm{J} = 47\,\mathrm{TW}$ from nuclear decays in the crust;[3] we see that this value is completely negligible compared to the energy incident from the sun, so just take $P_\mathrm{S} + P_\mathrm{J} \approx P_\mathrm{S}$. The Earth must radiate energy with equal power to be in thermodynamic equilibrium. We will assume that the Earth radiates as a black body. However, we will calculate a constant $\varepsilon$ that shall represent the inaccuracy of this assumption. Thus, the energy balance using the Stefan-Boltzmann relation will be as follows

$$P_\mathrm{S} = \varepsilon \sigma S T^4\,,$$

where $\varepsilon$ represents the calculated emissivity of the Earth, $\sigma = 5.67 \cdot 10^{-8}\,\mathrm{W{\cdot}m^{-2}{\cdot}K^{-4}}$ is the Stefan-Boltzmann constant, $S$ is the surface of the Earth, and $T = 13.7\,\mathrm{^\circ C}$ is the average surface temperature. Substituting for the Earth's surface $S = 4\pi R^2$, where $R = 6378\,\mathrm{km}$ we get the relation for emissivity

$$\varepsilon = \frac{P_\mathrm{S}}{4\pi R^2 \sigma T^4}\,,$$

which after adding the numerical values gives us $\varepsilon \approx 0.88$.

**A bit of thermodynamics** When doing any work, waste heat is generated. Even producing electricity, which we later use, produces waste heat. Eventually, after enough time has elapsed, any work done will convert to heat. Thus, in the long term, we are only interested in how much extra power the Earth can radiate to keep its surface temperature from rising by 2 °C. This total power will consist of the waste heat from generating electricity and from doing work that represents the thermal energy produced to generate electricity in the reactor.

[2] https://www.climate.gov/news-features/understanding-climate/climate-change-global-temperature
[3] https://en.wikipedia.org/wiki/Earth%27s_internal_heat_budget

**The amount of extra heat the Earth can radiate** We shall now estimate how much more heat output the Earth would radiate if its surface was heated by 2 °C. To calculate this, we modify the formula used in the section on the Earth's energy balance as follows

$$P + P_\mathrm{S} = 4\pi R^2 \sigma\varepsilon \left(T + 2\mathrm{^\circ C}\right)^4 \ ,$$

where $P$ is the extra thermal power that the Earth would radiate. By expressing it we get

$$P = 4\pi R^2 \sigma\varepsilon \left(T + 2\mathrm{^\circ C}\right)^4 - P_\mathrm{S} \,,$$

and after substituting and quantifying $P = 4\,600\,\mathrm{TW}$. In 2021, the total energy consumption on Earth was 176 000 TWh[4], which represents an average output of 20 TW over the course of a year. As we can see, global warming will not be caused by producing too much energy, but rather by the way of the production and by what we emit in the process.

**Second Law of Thermodynamics or something about efficiency** The Second Law of Thermodynamics states that the highest efficiency that a thermal machine can have between a hotter thermal reservoir with a temperature $T_\mathrm{H}$ and a cooler thermal reservoir with a temperature $T_\mathrm{S}$ is

$$\eta = \frac{T_\mathrm{H} - T_\mathrm{S}}{T_\mathrm{H}} \,,$$

which gives the efficiency of the Carnot cycle. The Second Law of Thermodynamics rules out the existence of a perpetual motion machine of the second kind, where all thermal energy would be converted to mechanical energy. Thus, our thermal power plants cannot have a higher efficiency than the relation above gives.

**Useful work we can do** A nuclear, geothermal, or even fusion power plant uses a steam turbine to generate mechanical and, consequently, electrical energy. Superheated steam under high pressure flows through the turbines and pipelines. The material limits on the temperature of such steam are about 650 °C. Consider that the steam coming out of the turbine is 100 °C. Then, for an upper estimate of the turbine efficiency using the Second Law of Thermodynamics, we get approximately 60 %. This estimate is too high, while conventional turbines have efficiencies around $40 - 50\%$ (combined cycle power plants can have efficiencies slightly over 60 %), but let's be optimistic.

So, with a power generation efficiency of 60 %, we can produce 2 800 TW of electricity on Earth over the long term. If we use electric motors with 90 % efficiency to do the work, we can do useful work with 2 500 TW.

### *Production of necessary energy*

How can we ensure such an energy consumption? We said in the introduction that solar panels are not suitable for this. The total output of 4 600 TW makes up about 3 % of the total solar energy that hits the Earth. Considering that 2/3 of the surface is comprised of oceans, we would have to replace almost a tenth of the surface with solar panels (at least two to three times more if we consider that some of the panels will be on the far side where it will be dark or will not produce power as efficiently due to the weather), which would probably not be entirely

[4] `https://ourworldindata.org/energy-production-consumption`

appropriate for the biosphere. It is important to note that utilizing this energy source would not entirely heat up the Earth. This is because only a portion of the solar energy would first be converted into electricity and then into work, as opposed to the conventional method, where it is converted directly into heat.

With current production by nuclear power plants and with current estimates of uranium deposits on the planet, this amount should be enough to give us 230 years.[5] However, with better technology, this time could be doubled. If we use fast reactor types that can recycle used fuel, the uranium supply could last us up to 30 000 years, at current energy consumption and production. The same source claims that the current annual energy production of nuclear power plants is 2 800 TWh. Thus, in the optimistic model, we have $84 \cdot 10^6$ TWh of energy stored in the uranium, which would last about two years at maximum possible consumption. There is also a large amount of uranium in the seas, which we currently cannot extract, but it looks like fission will not be enough to do the job. But we could call on a more powerful sibling to help; nuclear fusion.

**Current fusion options** Although we are still quite far away from a working fusion power plant, we can deal with the theoretical concept of one in this problem. In the first generation of fusion reactors, deuterium and tritium are assumed to be the fuel in a $1:1$ ratio. In one such collision, 17.6 MeV is released. Deuterium can be obtained cheaply from seawater, but tritium is more of a problem; we will need lithium to produce it.

**Quantity of lithium and deuterium** On Earth, there is $1.338 \cdot 10^9\,\mathrm{km^3}$ of water in the oceans.[6] The density of seawater is variable but assume the average density to be under $1020\,\mathrm{kgm^{-3}}$. Seawater also contains various soluble salts (in addition to the water itself). Let us assume that the amount of dissolved salts averages 4% of the mass of water. Based on this assumption, we can estimate that the mass of water in the Earth's oceans is $1.35 \cdot 10^{18}$ t. The molar mass of water is $18\,\mathrm{g{\cdot}mol^{-1}}$, and the hydrogen in water is $2\,\mathrm{g{\cdot}mol^{-1}}$ (note that a water molecule contains two hydrogen atoms). Thus, the molar mass ratio of hydrogen in water is $2:18$, and we get $\approx 1.51 \cdot 10^{17}$ t for the mass of hydrogen in the oceans. Deuterium makes up approximately 0.031 % of the mass of all hydrogen in the oceans,[7] so our estimate of deuterium on Earth is $4.7 \cdot 10^{13}$ t.

The total amount of lithium in deposits on Earth is estimated to be $89 \cdot 10^6$ t, with $22 \cdot 10^6$ t recoverable.[8] In addition, it is still found in significant quantities in seawater. We will assume that we can mine all the lithium in the deposits, giving us $89 \cdot 10^6$ t of lithium.

Lithium occurs in nature in two isotopes, namely $^6$Li and $^7$Li. Both isotopes can be used to produce tritium (both give tritium in a metabolic ratio of $1:1$ from lithium), but $^6$Li is more suitable for it. This isotope makes up 4.85% of the total amount of lithium, which, if converted, is $4.3 \cdot 10^6$ t, theoretically suitable for use in fusion.[9]

[5] https://www.scientificamerican.com/article/how-long-will-global-uranium-deposits-last/
[6] https://www.usgs.gov/special-topics/water-science-school/science/how-much-water-there-earth
[7] https://en.wikipedia.org/wiki/Deuterium
[8] https://bettermeetsreality.com/how-lithium-is-left-in-the-world-will-we-run-out-what-happens-if-we-do/?utm_content=expand_article
[9] https://en.wikipedia.org/wiki/Lithium#Occurrence

**Energy hidden in fusion** At first glance, the limiting factor for fusion will be lithium (there is significantly less of it, which means less tritium). We already have everything we need to estimate the energy with the relation

$$E = E_1 N = E_1 N_A \frac{m_{\mathrm{Li}}}{M_{\mathrm{Li}}} \,,$$

where $E$ is the total energy, $E_1 = 17.6\,\mathrm{MeV}$ is the energy released from one fusion of deuterium with tritium, $N$ is the number of particles we have (recall that the limiting factor is lithium), $N_A = 6.022 \cdot 10^{23}\,\mathrm{mol^{-1}}$ is Avogadro's constant, $m_{\mathrm{Li}} = 4.3 \cdot 10^{6}\,\mathrm{t}$ is the mass of $^{6}\mathrm{Li}$ and $M_{\mathrm{Li}} = 6.02\,\mathrm{g{\cdot}mol^{-1}}$ molar mass of $^{6}\mathrm{Li}$. Substituting $E = 7.57 \cdot 10^{36}\,\mathrm{MeV} \approx 336 \cdot 10^{6}\,\mathrm{TWh}$, which would last for 8.3 years to do the necessary amount of work.

If we decided to use the slightly worse $^{7}\mathrm{Li}$, we would substitute the total mass of lithium for $m_{\mathrm{Li}}$ and the relative molar mass of lithium for $M_{\mathrm{Li}}$, $6.94\,\mathrm{g{\cdot}mol^{-1}}$. In this case, $E = 1.36 \cdot 10^{38}\,\mathrm{MeV} \approx 6.04 \cdot 10^{9}\,\mathrm{TWh}$, which would last for 24 years of doing the work.

*Conclusion*

In theory, we have the potential to generate up to 2 500TW of power in the long term without causing harm to the climate. However, given our current technological capabilities, we would not be able to produce enough energy to do the work for more than two centuries.

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FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.