Problem V.E ... disappearing CD

12 points; průměr 10,15; řešilo 46 studentů

Use diffraction on the grating to determine the density of data written to the CD. Try to compare the results with the DVD. *Káťa still has a lot of CDs at home. Pepa envies her.*

Theory

You have probably heard that light is electromagnetic radiation and that it, therefore, behaves like a wave. One of the consequences of this wave behavior is a phenomenon called diffraction.

Let's take a coherent light source, in this case, a laser, and shine it on a so-called diffraction grating – a thin film containing periodically repeating slits, allowing the light to pass through more easily. Most of the radiation passes through without changing direction, but some of it is deflected by diffraction and forms an interference pattern on the screen due to interference. We should see an odd number of dots of light, technically called maxima, in a line where the middle dot (the central maximum) is the brightest – corresponding to the passage of light without interference. Symmetrically with respect to this center, we see other maxima.

We calculate their position using the following formula[1]

$$\sin\theta_k = \frac{k\lambda}{b}\,,$$

where θ_k is the angle of deviation of the kth maximum from the direction of the central maximum, λ is the wavelength of the light, and b is the distance between the slits [2]. Since the sine of any angle is always less than or equal to one, we cannot choose an arbitrarily large k –we get a finite number of maxima.

Since we will not directly measure the angle θ_k but the distance of the kth maximum from the central maximum, we will calculate this angle from the measured distances as

$$\theta_k = \arctan\frac{d_k}{L}\,,$$

where d_k is the distance of the kth maximum from the central maximum and L is the distance of the screen from the diffraction grating. After substitution, we get the compound function $\sin(\arctan(x))$. In general, if we compose some goniometric function with some inverse goniometric function, the notation can be simplified. In our case, we can deduce the following relation

$$\sin x = \tan x\cos x = \frac{\tan x}{\frac{1}{\cos x}} = \frac{\tan x}{\sqrt{\frac{1}{\cos^2 x}}} = \frac{\tan x}{\sqrt{\frac{\sin^2 x+\cos^2 x}{\cos^2 x}}} = \frac{\tan x}{\sqrt{\frac{\sin^2 x}{\cos^2 x}+1}} = \frac{\tan x}{\sqrt{\tan^2 x+1}}\,.$$

[1] This formula is valid, only if the laser beam is perpendicular to the diffraction grating

[2] To give you an idea, the typical magnitude of b of a diffraction grating is in micrometers scale.

By substituting this relation, we get the final form of the formula that we will use to calculate b

$$\sin \arctan \frac{d_k}{L} = \frac{k\lambda}{b}\,,$$

$$\frac{\frac{d_k}{L}}{\sqrt{\left(\frac{d_k}{L}\right)^2 + 1}} = \frac{k\lambda}{b}\,,$$

$$b = k\lambda\sqrt{1 + \left(\frac{L}{d_k}\right)^2}\,.$$

Finally, a little more about why CDs and DVDs behave like diffraction gratings. On the surface of the CD and the DVD, underneath the protective layer, there are regularly arranged irregularities that, when read by an optical drive, the computer can reconstruct the stored data. Because of these regular bumps, the CD and DVD behave as a diffraction grating, and we are able to determine the dimension of these irregularities, i.e., the density of information writing.

Measurements

First, we tried to separate the transparent protective part from the two discs carrying the information. Since we were unable to do this in the case of the CD, we had to choose two different measurement procedures.

Measurement of DVD In the case of the DVD where we managed to separate the transparent part, we proceeded as follows. We glued the DVD to the edge of the table and placed the laser pointer on the table. It shined a green light of approximate wavelength 532 nm. Since it didn't shine straight ahead, it had to be taped with duct tape to make it shine in the same spot all the time. We used a cabinet distant (78.5 ± 0.5) cm as a screen. When we shined the light on it, we saw the interference pattern described in the theoretical section. In total, three maxima were seen, but only two fit on the cabinet. The laser needed to shine perpendicularly to the surface of the DVD. We did this by attaching the laser to a metal part of the table to ensure sufficient perpendicularity for our needs.

After lighting the laser, we marked the position of the next maximum with a pencil and turned the DVD so we wouldn't keep shining it in the same spot. We did this for a total of ten times. Since we had fixed the laser pointer well, we needed to mark the central maximum just once, as it was in the same spot every time. Finally, we used a tape measure to determine the distance between the minor maxima and central maximum.

Measurement of CD Since we could not separate the protective layer in the CD, we had to choose a different procedure. We mounted the laser vertically so that it shined as perpendicular to the CD placed on the table as possible. Again, we rely on the geometry of the furniture to ensure sufficient perpendicularity. The beam on the surface of the CD diffracts, bounces, and forms an interference pattern on the shelf (99.0 ± 0.5) cm above the CD. Since the CD has a lower writing density, there were a total of five interference maxima and closer to each other.

During the measurements, we followed a similar procedure as for the DVD. For each measurement, we took the position of the first and second maxima and rotated the CD. This way, we repeated the process five times – so we took ten measurements. Again, we made sure that the position of the central maximum did not change. The distance of the secondary maxima from the central maximum was again measured with a tape measure.

Data

The laser pointer produced a diverging beam of approximately 0.5 cm in diameter when it hit the screen, so distances d_k are given with this precision. At the same time, we cannot be sure about the precision of the wavelength of the laser. Based on the manufacturer's data [3] we estimate the absolute deviation as $\Delta\lambda = 10\,\mathrm{nm}$.

Values of DVD The measured data are listed in table 1.

Tab. 1: Measured distances of the 1st maximum d_1 of the interference on the DVD.

N	$\frac{d_1}{\mathrm{cm}}$
1	82.0
2	81.0
3	81.5
4	83.0
5	83.0
6	81.0
7	81.0
8	81.5
9	78.0
10	77.5

After calculating the mean and the standard deviation, we find distance $d_1 = (81.0 \pm 1.8)$ cm. The value of the spacing between the slits for DVD and its absolute deviation is calculated using the formula at the end of the theoretical section.

$$b_{\mathrm{DVD}} = k\lambda\sqrt{1+\left(\frac{L}{d_k}\right)^2}\,,$$

$$\Delta b_{DVD} = \sqrt{\left(\frac{\partial b_{\mathrm{DVD}}}{\partial \lambda}\right)^2 \cdot \Delta\lambda^2 + \left(\frac{\partial b_{\mathrm{DVD}}}{\partial d_1}\right)^2 \cdot \Delta d_1^2 + \left(\frac{\partial b_{\mathrm{DVD}}}{\partial L}\right)^2 \cdot \Delta L^2} =$$

$$= \sqrt{k^2\left(1+\left(\frac{L}{d_k}\right)^2\right)\Delta\lambda^2 + \frac{(k\lambda)^2}{1+\left(\frac{L}{d_1}\right)^2}\frac{L^4}{d_1^6}\cdot\Delta d_1^2 + \frac{(k\lambda)^2}{1+\left(\frac{L}{d_1}\right)^2}\frac{L^2}{d_1^4}\cdot\Delta L^2}\,,$$

$$b_{\mathrm{DVD}} = (741 \pm 16)\ \mathrm{nm}\,.$$

Values of CD Here are the values from the second measurement listed in table 2.

[3] `https://www.avetech.cz/data/original/vario/7F31EC60-D261-4801-B59E-DCFD9A32FD1E_DB5CA8C0-F923-4E38-8E7A-DCDDFAE936A8.pdf`

Tab. 2: Measured distances of the 1st maxima d_1 and 2nd maxima d_2 of the interference on the CD.

N	$\frac{d_1}{\mathrm{cm}}$	$\frac{d_2}{\mathrm{cm}}$
1	38.5	106.5
2	38.5	107.0
3	39.0	106.0
4	38.5	106.5
5	38.0	105.5

From the measured data, we obtain in the same way

$$d_1 = (38.5 \pm 0.4)\ \mathrm{cm}\,,$$
$$d_2 = (106.2 \pm 0.8)\ \mathrm{cm}\,.$$

We plug these values into the formulas above to obtain the distance between the slits according to the first measurement b_{CD1} and according to the second measurement b_{CD2}, respectively.

$$b_{\mathrm{CD1}} = (1\,468 \pm 31)\ \mathrm{nm}\,,$$
$$b_{\mathrm{CD2}} = (1\,455 \pm 28)\ \mathrm{nm}\,.$$

Note that, according to theory, both values should come out the same because the distance between the slits does not depend on the order of the maximum.

Discussion

According to internet sources[4] is the writing density of the DVD $b_{\mathrm{DVD}} \doteq 740\,\mathrm{nm}$ and the writing density of the CD $b_{\mathrm{CD}} \doteq 1\,600\,\mathrm{nm}$. In the first case, the measured value falls within the error interval. In the second, it does not, but the difference is not very large. Still, we measured with a relatively small error. The explanation may be that there are several types of CD media: CD-ROM, CD-R, Audio CD, … and each of them can have slightly different writing density values. But it is still the same technology, so we can expect that these values will not be different by as much as an order of magnitude.

It is also worth mentioning that we have measured the density of circular traces into which the indentations are arranged. The writing density along these tracks is even higher and produced maxima, this time in the perpendicular direction to the first set. The latter, however, were far apart and we could not measure them.

The largest source of error was our inability to accurately determine the wavelength of the laser, which we could have solved by using a better laboratory laser. Another source of error is our inability to accurately determine the position of the interference maxima. In fact, the maxima were not unambiguous points, but spots of 0.5 cm in size. This problem could be solved by using a laser with a better-prepared beam.

[4] `https://www.nnin.org/sites/default/files/files/Karen_rama_TG_part2_0.pdf`

Conclusion

We measured the distance between the slits of the transverse writing on the DVD as $b_{\mathrm{DVD}} = (741 \pm 16)$ nm. The value for the CD medium was measured separately for the first and second maximum as $b_{\mathrm{CD1}} = (1\,468 \pm 31)$ nm a $b_{\mathrm{CD2}} = (1\,455 \pm 28)$ nm.

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