Problem IV.P ... the boat is sailing

10 points; průměr 5,65; řešilo 55 studentů

Discuss what physical phenomena affect the cruising speed of a ship and submarine. What resistive forces act on them? What is the highest cruising speed that a ship or submarine can sail?

Jindra went punting on the river Cam.

Introduction

When a ship or submarine is sailing,[1] two main things happen. Its engines use the propellers to produce a pulling force which accelerates the ship forwards, and at the same time the environment (particularly water) exerts a resistive force against this movement. The maximum possible speed is reached when both forces balance each other out. Let's take a look at the drag force in fluids, its causes and its subtypes.

Resistive forces

Unlike regular friction, drag force in fluids depends on the magnitude of velocity either of the moving body relative to the fluid, or of two layers of fluid relative to each other. At low velocities, laminar flow occurs, in which case the resistive force is linearly dependent on the velocity. When the velocity exceeds a certain critical value, turbulent flow occurs, which means that the drag force is now proportional to the square of velocity. We can use the Reynolds number Re to determine whether we are dealing with laminar or turbulent flow.

$$Re = \frac{\rho v L}{\eta}$$

where ρ is the density of the fluid, v is the relative velocity of the body, L is the characteristic linear dimension and η is the dynamic viscosity of the fluid. When $Re < Re_K$, the flow is laminar, and if $Re > Re_K$, the flow is turbulent. The value Re_K is usually determined experimentally for the geometry of a given system.

With laminar flow, layers of the fluid do not mix. In the case of turbulent flow they do mix as vortices appear. Laminar and turbulent flow can even be distinguished by the naked eye, since laminar flow appears stationary (flowing fluid appears as if it was just a photograph) but turbulent flow changes shape. From experience, we know that motion of a ship or a submarine involves turbulent flow.

Classification of drag forces

Drag forces exerted by a fluid environment can be divided based on their cause into two categories: pressure (form) drag and viscous drag.

Form drag is related to the shape of the body which pushes aside the particles of the fluid and changes their direction of motion.

Viscous drag is caused by the friction between the fluid and the surface of the body. When the body comes into contact with the fluid, an infinitesimally thin layer of fluid on the surface does not move relative to the body (the body "drags" this layer along), which causes friction, since the other layers are moving. For slow ships, this is approximately 80% of the total drag, for fast ships it's approximately 50%.

[1] TL note: While submarines and some ships do not use sails, this is the commonly used expression.

Calculating the resistive force

The net drag force is the sum of above-mentioned effects. For example, for fluid flowing around a ball, approx. 90 % of the net resistive force is caused by pressure drag, while for the flow around a wing it is only around 10 %. Assuming the whole body is submerged in the fluid, the drag force F_D can be calculated as

$$F_\mathrm{D} = \frac{1}{2}\rho v^2 C_\mathrm{D} S$$

where ρ is the density of the fluid, v is the velocity of the body relative to the fluid, S is the effective cross sectional area (perpendicular to the direction of motion), and C_D is the drag coefficient which depends on geometry of the situation, and is determined experimentally (for the most common shapes, it can also be found on wikipedia).

Further classification of pressure drag

The magnitude of pressure drag depends on multiple effects which we will be looking at in the next part. These effects can contribute to viscous drag as well, but they are all caused by the geometry of the body, and as such are considered to be a type of pressure drag.

Drag by wave formation This type of resistance caused by the environment only affects surface vessels and corresponds to the energy required to move water away from the hull of the ship. This energy is expended on forming a wave (which also carries the energy away from the ship). The strongest waves are produced by the front and rear ends of the ship, and these waves are divided into divergent and transverse waves.

Transverse waves make up the majority of this drag, since they affect how much of the surface of the ship is in contact with water (viscous part of drag). The wavelength of transverse waves is proportional to the cruising speed of the ship, and at some value (so-called hull speed) this wavelength is equal to the length of the ship (different ships can have different hull speeds). In this case, the area of the ship's hull which is in contact with water is the greatest, which causes the maximum value of viscous drag (first maximum of drag). If we further increase the cruising speed (to the so-called hump speed), the wavelength of transverse waves increases to 1.5 times the length of the ship. This causes the water level at the prow to decrease and causes the ship to tilt backwards, giving an impression that the ship is always moving "uphill". Ultimately, this increases the energy required to maintain the given cruising speed (the second maximum of drag). With further increases in cruising speed, drag by wave formation decreases slightly (however, we need to overcome the mentioned barriers).

The angle between divergent waves on opposite sides of the ship is 39 ° (independent of the cruising speed) and they can be seen as outer edges in the wake of the ship.

Drag by lift While wave formation causes drag against the direction of motion, drag caused by lift has a component perpendicular to the direction of motion, which can be used e.g. in aviation. This force is a consequence of Newton's law of action and reaction, where the hull of a ship or of a wing of a plane changes the direction of motion of particles of the fluid downwards, and so a reaction force acts on it upwards (dynamic lift).

Drag caused by crossing the sonic barrier Exceeding the speed of sound in a given environment causes shock waves, which leads to further loss of energy. The magnitude of the resulting drag force does not depend on viscosity. Since neither ships nor submarines are able to move that fast (the speed of sound in water is approximately $1\,500\,\mathrm{m\cdot s^{-1}}$), we will disregard this effect from now on.

Power of drag forces and maximum cruising speed

With ships, it is typical to work not with the pulling (tractive) force of the engines, but with their power. For this reason, it would be useful to describe the lost energy as power loss due to resistance of the environment. Then the maximum cruising speed is the speed for which the power produced by the engines balances out the power loss. We obtain the power loss P_D by multiplying the drag force F_D by the cruising speed v.

$$P_\mathrm{D} = F_\mathrm{D} v = \frac{1}{2}\rho v^3 C_\mathrm{D} S$$

In equilibrium, the power of the vessel's engines is $P = P_\mathrm{D}$ and the maximum cruising speed v_max satisfies the condition

$$v_\mathrm{max} = \sqrt[3]{\frac{2P}{\rho C_\mathrm{D} S}}\,.$$

This description is particularly useful if the vessel does not produce waves (submerged submarine), but it is not sufficient for calculating the maximum cruising speed of a ship on the surface, since it does not involve the drag force caused by wave formation.

Accounting for the influence of waves or Crouch constant

George Crouch was an American engineer who lived at the turn of the 19th and 20th centuries and became famous as a designer and builder of high-speed boats, which often won the Gold Cup. His formula for the maximum velocity of a ship is:

$$v_\mathrm{max} = C\sqrt{\frac{P}{M}}\,,$$

where C is the Crouch constant (which depends on the type of the boat), P is the power of the boat's engines and M is its mass. The main disadvantage of this formula is the fact that Mr. Crouch did not know the metric system, so he expressed power P in horsepower ($1\,\mathrm{kW} = 1.34\,\mathrm{hp}$), mass M in pounds ($1\,\mathrm{kg} = 2.2\,\mathrm{lbs}$) and velocity v_max in miles per hour ($1\,\mathrm{mph} = 1.61\,\mathrm{km\cdot h^{-1}}$). In the table below, we list the values of Crouch constant for some types of boats.

Tab. 1: Crouch constant for various types of boats

Boat type	Crouch constant
cruiser, regular "runaboat", cruise ship	150
light high-speed cruiser, high-speed runaboat	190
racing boat	210
hydroplane	220
racing catamaran and "sea sled"	230

Minimizing drag forces

For ships

The more suitable the shape of the ship, the larger its Crouch constant is. There are multiple ways to achieve this and various types of ships are modified in different ways according to their purpose.

For large and slow ships with large draught (submerged size), we try to minimise the drag due to wave formation, since form drag could only be decreased by decreasing the draught, which would negatively affect the load it could carry. In order to do that, a cylinder-like protrusion (bulbous bow) is usually added to the front of the ship underneath the waterline. When it is at an optimal distance from the hull, this protrusion can create another transverse wave, which cancels out with the usual transverse wave formed at the prow via destructive interference, thus decreasing the drag.

The other way to decrease drag is through minimising the size of the submerged part of the ship (minimising viscous drag). This is used especially in the case of smaller and faster ships. To achieve this goal, dynamic lift is utilised (as it is produced for a suitable shape of the body of the ship), which helps the static lift (buoyancy, described by Archimedes' law) in overcoming the weight of the ship. When the cruising speed is higher, it produces a greater dynamic lift, which decreases the required static lift and the ship can rise higher in the water (and that decreases the surface slowed down by water). In practice, we can see it with small motorboats, which "fly" across the surface rather than sail across it at high speeds.

The fastest ship should have a good dynamic lift (included in the Crouch constant), low mass and high-power engines (the mass of an engine is related to the power it can produce, so we need to look for a compromise).

For submarines

With submerged submarines, we are trying to minimise the drag coefficient C_D. After a brief look in the physics tables, we find a shape similar to a raindrop, which has $C_\mathrm{D} = 0.04$. Looking at modern military submarines, we can notice that they indeed have this elongated raindrop-like shape. Thus we optimised the shape, another important factor is size. We are trying to minimise the effective surface area of the submarine. The third important factor is power, since a submarine with more powerful engines moves faster (here we again need to look for a suitable compromise, since engine power is related to size).

Specific estimates of maximum cruising speed

Racing ship

Let's choose a ship with a suitable shape (for example a sea sled) and consider the value of the Crouch constant to be $C = 230$. For a given shape, linearly scaling the dimensions of the ship will result in a cubically scaling of its mass, so it makes sense to choose the smallest, one-person ship. There must be someone manning the ship, we will assume it is a person with a mass of 80 kg. An important factor is the choice of the engine, we'll look for those with good power-to-mass ratio. Of course, the ship we design needs to have sufficient lift to carry all the required components.

To simplify the calculations, we will assume the body of the ship is shaped like a cuboid (which a sea sled indeed resembles, with minor differences), and that the hull of the ship is

0.5 cm thick, and it is made from a light and strong material, e.g. aluminium. The original dimensions of a sea sled are: length 838 cm, width 274 cm and draught 30 cm (we will consider the total height of the ship to be 50 cm), which gives the volume of the sumberged part 6.9 m^3. Since a sea sled isn't exactly a cuboid, we will estimate that the submerged volume for a real ship is 60 % of this value, which is $V_\mathrm{p} = 4.1\,\mathrm{m}^3$. At rest, the forces due to gravity and buoyancy must balance each other out. This allows us to find the maximum mass of the ship m_max, which gives:

$$m_\mathrm{max} g = F_\mathrm{g} = F_\mathrm{vz} = V_\mathrm{p} \rho g\,.$$

We'll consider the density of water to be $\rho = 1\,000\,\mathrm{kg{\cdot}m^{-3}}$ and divide the equation by acceleration due to gravity g. After plugging in the numbers, we get $m_\mathrm{max} = 4\,100\,\mathrm{kg}$.

Let's estimate the mass of the hull with a thickness of 0.5 cm. First we estimate its volume by subtracting the volume of a cuboid with the above mentioned dimensions and the volume of another cuboid with dimensions decreased by the thickness of the hull (length 837 cm, width 273 cm and height 49.5 cm). The volume of the hull is $V_\mathrm{pl} = 0.17\,\mathrm{m}^3$, the density of aluminium is $\rho_\mathrm{Al} = 2\,700\,\mathrm{kg{\cdot}m^{-3}}$, the mass is therefore $m_\mathrm{Al} = \rho_\mathrm{Al} V_\mathrm{pl} = 460\,\mathrm{kg}$.

We still need to strengthen the body of the ship somehow, add a battery for starting, electronics, a control mechanism, a fuel tank and other support systems. Let us assume that all these things combined will not weigh more than 1 000 kg.

Finally, we only need to choose a suitable engine. A perfect candidate is an outboard motor with power of almost 450 kW and mass 570 kg. After subtracting the mass of a person, the hull and other systems, we find out that the ship can still carry an extra load of 2 500kg, which lets us use 4 such engines with total power 1 800 kW and total mass 2 280 kg.

Now we know everything necessary to estimate the maximum cruising speed. The total mass of our ship is 3 820 kg = 8 420 lb and its motors have power 1 800 kW = 2 400 hp. After plugging into Crouch's formula, we get $v_\mathrm{max} = 123\,\mathrm{mph} = 200\,\mathrm{km{\cdot}h^{-1}}$. This speed record was indeed reached by Gary Wood in 1932. The absolute cruising speed record on water is currently held by Ken Warby, who reached a cruising speed of almost $560\,\mathrm{km{\cdot}h^{-1}}$ on a ship with jet engines (we must note that mortality of pilots of such high-speed ships when attempting to surpass this record is around 60 %).

Racing submarine

In this case, we focus on the diameter of the widest part of the submarine, which we need to choose in such a way that one person can fit inside. Let us assume that average human height is 180 cm and that such a person is sitting while steering the submarine. In that case, the maximum thickness of the submarine could be 2 m, which should be sufficient for comfortably sitting in the middle part (the length is then approx. 6 m).

As an engine for our submarine, it is advantageous to use an electric motor with a sufficient supply of batteries, since it does not have access to air for combustion. Commercially, a direct current engine with power between 20 kW and 2 000 kW can be obtained. The manufacturer states that the dimensions of such an engine do not exceed 60 cm, which is good for us, since we can fit it inside the submarine. We only need to estimate how many batteries we need and how much energy we can store in them, based on which we choose the power of the engine (so that the submarine could move longer than a few minutes). One candidate for battery type has capacity 1.3 kWh and dimensions $(26 \times 17 \times 23)$ cm according to the manufacturer (the volume of one such battery is cca $10\,200\,\mathrm{cm}^3$).

Now we estimate the volume of the submarine. A raindrop-like shape with diameter 2 m and length 6 m is approximated by a cylinder with diameter 1.5 m and length 5 m, which has a volume of approx. $8.8\,\mathrm{m^3}$. Let's assume that in order to sit comfortably, a human needs a cuboid with height 2 m, and base with width 60 cm, and length 70 cm, which gives a volume of $0.84\,\mathrm{m^3}$. The largest dimension of the motor is 60 cm, so its maximum volume can be approximated by a cube with this side length, whose volume is $0.216\,\mathrm{m^3}$. Besides that, the submarine needs to be equipped with some support systems (steering, ballast tanks, life support, computers, cooling etc.), but let us assume that all those systems combined do not take up more than $2\,\mathrm{m^3}$. After subtracting all the mentioned things from the volume of the submarine, $5.7\,\mathrm{m^3}$ are left for batteries, which corresponds to 558 above-mentioned batteries. However, they can't be placed exactly next to each other in the whole volume, and the submarine also gets quite thin in the rear, so we will estimate that 20 % of this initial estimate of the number of batteries do not actually fit in the submarine, and that leaves 446 batteries with total capacity cca 580 kWh.

Next, let us assume that 10 % of the stored energy is consumed by support systems of the submarine, so the electric motor is able to use 522 kWh. If we choose an engine with power 1000 kW, the submarine can sail for half an hour, which should be sufficient for testing its speed. Let us assume that the motor has efficiency 90 %, so the effective power is 900 kW. We calculate the effective cross-sectional area using the known diameter as $S = \pi d^2/4$, where d is the diameter of the thickest part. After plugging in the numbers, $S = 3.14\,\mathrm{m^2}$. We will consider testing in freshwater with density of $1\,000\,\mathrm{kg{\cdot}m^{-3}}$. We now know all the data needed to estimate the maximum cruising speed, after plugging in the numbers: $v_{\mathrm{max}} = 24.3\,\mathrm{m{\cdot}s^{-1}} = 87.5\,\mathrm{km{\cdot}h^{-1}}$. For comparison, the speed record of a submerged submarine is held by the Soviet submarine K-222 with maximum reached cruising speed $83\,\mathrm{km{\cdot}h^{-1}}$, so our approximate description works pretty well.

Afterword

We must note that the models considered above are only rough estimates to get a basic idea, since we did not consider many factors, so do not try to verify the results at home experimentally.

sources

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