Problem IV.E ... I will hang it

12 points; průměr 8,64; řešilo 45 studentů

We have a rope wrapped around a bar with a weight of mass m at one end. Measure the dependence of the mass of the weight M at the other end, needed to set the rope in motion, on the number of times the rope wraps around the bar.

Patrik thinks about different methods of... calculation.

General knowledge

To determine the coefficient of friction, we will use the so-called capstan equation, which describes the tension F on a cylindrical surface

$$F = F_0 \mathrm{e}^{f\varphi} , \tag{1}$$

where φ is the rope wrap angle, f denotes the coefficient of the static friction between the rope and a rounded surface, and F_0 corresponds to the input tension. The derivation of the equation(1) can be found in the older problem rope.[1]

In our case, F_0 will be mediated by the gravitational force given by the action of our chosen weight with mass m, and F will be the gravitational force of the weight M for different angles of rotation φ. To be able to determine the final correlation, we will only need to know the masses of the weights needed to put the rope into the motion

$$M(\varphi) = m\mathrm{e}^{f\varphi} .$$

Measurement

Measuring equipment

During the experiment, we have used a weight with a mass of $m = (0.500 \pm 0.001)\,\mathrm{kg}$, a plastic rope with a diameter of $d = 2\,\mathrm{cm}$ with a total mass of $m' = 25\,\mathrm{g}$ and a steel hanging bar for the doors with a diameter of $D = 10\,\mathrm{cm}$.

Measurement procedure

We have attached a weight with a mass m to one of the rope's ends; next, we have attached a water container with a mass $m'' = (0.325 \pm 0.001)\,\mathrm{kg}$ to the other end of the rope. We always winded the rope around the bar by a required number of revolutions and then gradually added water to the container until the rope started to move spontaneously. We have taken one measurement for each turn. The mass of the water container was measured on a kitchen scale with an accuracy of 1 g, and the last measurement was done on a personal scale because of the large mass of weight.

Measured values and results processing

The measured data are written in Tab. 1. Next, we plotted the data in a graph in Fig. 1, and then we used software `gnuplot` to fit them with an equation in the form

$$y(x) = A\mathrm{e}^{Bx} , \tag{2}$$

[1] `https://fykos.org/_media/rocnik32/ulohy/pdf/uloha32_6_4.pdf`

where $A = (0.58 \pm 0.05)\,\mathrm{kg}$ and $B = (0.379 \pm 0.015)\,\mathrm{rad}^{-1}$ are the fitting parameters.
We determined the uncertainty quoted for the mass measurements from the added volume of water, which corresponded to the section when the rope began to unwind spontaneously under the container's weight.

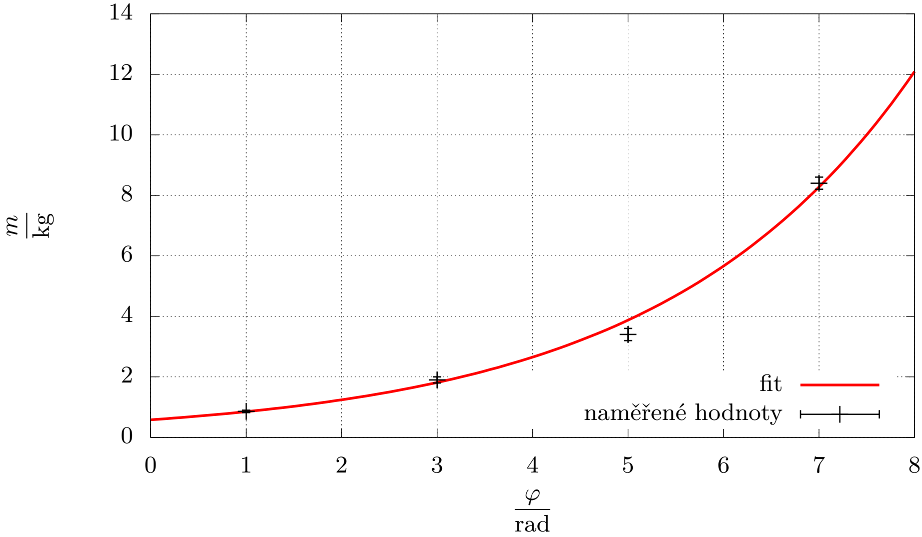

Fig. 1: Graph of the dependence of the angle φ on the mass m.

Tab. 1: Dependence of the mass required to set the rope in motion depending on the wrap angle.

$\frac{\varphi}{\mathrm{rad}}$	$\frac{m}{\mathrm{kg}}$
π	0.86 ± 0.04
3π	1.90 ± 0.10
5π	3.40 ± 0.20
7π	8.40 ± 0.20

Discussion

From the graphed equation (3) in Fig. 1, we can see that we have correctly determined the initial weight M and also the sought-after friction coefficient $f = (0.379 \pm 0.015)$. We could have neglected the rope's mass m' in the calculations due to the relatively large mass of the weight M and thanks to the fact that the rope is mostly wound around the bar.

In the experiment, we measured the exponential dependence. Thanks to its shape, it is usually difficult to measure a large number of values because the required mass to get the rope to the motion grows exponentially; thus, the force needed to exceed the frictional force may be even higher than the maximum load capacity of the rope. From the graph in Fig. 1, we can see that to wind $\varphi = 9\pi\,\mathrm{rad}$, given the investigated dependency, we would need to hang a weight with a mass of approximately $20\,\mathrm{kg}$ to unwind the rope, which would probably be beyond the strength of the used rope. At the same time that we were putting weight onto the rope, it was being deformed; however, this should have minimal effect on the dependency we are investigating because the part winded around the bar retains its initial length. The statistical uncertainty of individual data points could be reduced by increasing the number of measured values. The maximum possible number of wraps could be effectively increased by using lighter weights, but in this case, we would have to include the non-negligible mass of the rope.

During winding, we left only a small free space between each loop of rope to avoid mutual friction between the loops. The bar's surface was metal-plated so that its coefficient of friction would be relatively small during translation. We could obtain more accurate results using a force meter, thus avoiding uncertainty by pouring the water.

By comparing the equations (1) and (3), we can formally neglect the unit parameter of the fitted value B and set the coefficient of static friction between the rope and the bar to be $f = (0.379 \pm 0.015)$. By comparing the value of f with the tabulated value[2], we can say that its value is the closest to the value of the coefficient of friction of the steel and nylon combination $f' = 0.4$ which is commonly used for rope manufacturing.

Conclusion

We have studied the phenomenon of friction on cylindrical surfaces and examined the dependency of the mass of the load M needed to put the rope into motion on the number of turns around the bar. The measured values in Tab. 1 were graphed with the expected dependence from the capstan equation

$$y(x) = A\mathrm{e}^{Bx}\,, \tag{3}$$

where the parameters satisfy $A = (0.58 \pm 0.05)\,\mathrm{kg}$ and $B = (0.379 \pm 0.015)\,\mathrm{rad}^{-1}$. By formally neglecting the unit in the fitted parameter B, we have determined the coefficient of static friction between the rope and the bar to be $f = (0.379 \pm 0.015)$, which corresponds well enough to $f' = 0.4$ for a pair of nylon and steel surfaces in the 2σ criterion.

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[2] https://www.engineeringtoolbox.com/friction-coefficients-d_778.html