Problem IV.3 ... road closure

6 points; průměr 5,12; řešilo 91 studentů

We all know it – road closures and endless standing at traffic lights. The light is green for 60 s, *but by the time everyone gets going, it is red again. Consider the* 0.5 s *reaction time for a driver to get moving after the car in front of him has done so. By what percentage would the number of cars that pass through the closure increase if everyone in line started moving simultaneously? The first car stands at the traffic light level, the distance of the front bumpers of all cars is estimated to be* 5 m, *and they all accelerate uniformly for* 5 s *to a speed of* $30\,\mathrm{km{\cdot}h^{-1}}$, *with which they proceed further into the closure.*

They have been digging sewers in Jarda's village for three years now.

Let us first denote the quantities from the problem statement: the time when the light is green and the closure passable is $T = 60\,\mathrm{s}$, and the reaction time of the drivers is $t_\mathrm{r} = 0.5\,\mathrm{s}$. Moreover, the speed to which they accelerate is $v = 8.3\,\mathrm{m{\cdot}s^{-1}}$, the distance of the cars' front bumpers is $d = 5\,\mathrm{m}$, and we can calculate the acceleration as $a = \Delta v/\Delta t = 8.3/5 = 1.7\,\mathrm{m{\cdot}s^{-2}}$.

The problem statement asks us to find the ratio of cars that pass through the closure when everyone starts moving simultaneously and the case when it takes some time for drivers to react to the car in front of them accelerating away. Let us count the number of cars for both situations.

In the first case where everyone starts moving simultaneously, the cars can move for whole T seconds. Therefore, the maximal distance they can travel is

$$s = \frac{1}{2}a5^2 + (T-5)v \doteq 479\,\mathrm{m}\,.$$

Thus, we will get the distance from which it is still possible to reach the boom barrier before the red lights up. The distance of the "n"-th car from the boom barrier is given by

$$d_n = (n-1)d\,.$$

From condition

$$d_n \leq s\,,$$

we get that the last car to make it through the boom barrier is $n_1 = 96$ in the order.

The second case with the reaction time of the drivers only modifies (specifically reduces) the time that cars have to move. The amount of time that the n-th driver will have is "reduced" by a factor

$$t_n = (n-1)t_\mathrm{r}\,.$$

We can calculate the distance traveled by the n-th car as

$$s_n = \frac{1}{2}a5^2 + (T-5-t_n)v\,.$$

The limiting case to pass the boom barrier is now: $s_n \geq d_n$. Thus, for n we get

$$n = \frac{\frac{1}{2}a5^2 + (T-5)v}{d + vt_\mathrm{r}} + 1\,.$$

For the given values, this means that $n_2 = 53$ cars will be able to pass through the boom barrier. The ratio we are looking for is $n_1/n_2 \doteq 1.8$. Thus if all drivers started simultaneously, roughly 80 % more cars could pass the boom barrier.

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FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of MFF UK, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.