Problem IV.2 ... frozen balloon

3 points; průměr 2,24; řešilo 91 studentů

A balloon of mass $m_\mathrm{b} = 2.7\,\mathrm{g}$ and volume $V_0 = 4\,\mathrm{l}$ was filled with helium of the same temperature as the surrounding air, i.e., $T_0 = 20\,^\circ\mathrm{C}$. Inside the balloon, the pressure is $\Delta p = 2\,\mathrm{kPa}$ higher than in the surrounding area. To what temperature do we need to cool the balloon and the gas in it so it stops floating? Assume that there will be atmospheric pressure in the balloon after cooling down. *Vojta trades balloons for inspiration.*

Firstly, we need to determine the mass of helium in the balloon. In order to do that, let us consider the equation of state

$$pV_0 = \frac{m}{M_\mathrm{He}} RT_0 \quad \Rightarrow \quad m = \frac{(p_\mathrm{a} + \Delta p) V_0 M_\mathrm{He}}{RT_0}\,,$$

where p_a is the atmospheric pressure and M_He is the molar mass of helium. We will now denote $\rho_\mathrm{v} = 1.204\,\mathrm{kg{\cdot}m^{-3}}$ as the density of air at temperature T_0 and at atmospheric pressure. We can determine the condition for the balloon to stop hovering from the equilibrium of the buoyant force and gravitational forces. We can formulate this into an inequality

$$V\rho_\mathrm{v} g < (m + m_\mathrm{b}) g \quad \Rightarrow \quad V < \frac{1}{\rho_\mathrm{v}} \left(\frac{(p_\mathrm{a} + \Delta p) V_0 M_\mathrm{He}}{RT_0} + m_\mathrm{b} \right)\,,$$

in which we will express the volume V from the equation of state as a function of the balloon temperature T as

$$p_\mathrm{a} V = nRT = \frac{(p_\mathrm{a} + \Delta p) V_0 T}{T_0} \quad \Rightarrow \quad V = V_0 \left(1 + \frac{\Delta p}{p_\mathrm{a}}\right) \frac{T}{T_0}\,,$$

which we can substitute into the inequality we derived above. Therefore, we will get

$$T < \frac{p_\mathrm{a}}{\rho_\mathrm{v}} \left(\frac{M_\mathrm{He}}{R} + \frac{T_0}{p_\mathrm{a} + \Delta p} \frac{m_\mathrm{b}}{V_0} \right) \doteq -71.5\,^\circ\mathrm{C}\,.$$

It is enough to submerge the balloon in liquid nitrogen, and it will stop hovering after a while.

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