Problem III.4 ... escape to Tau Ceti

7 points; průměr 3,17; řešilo 84 studentů

Since our Sun will explode one day, it will be necessary to organize the construction of an evacuation spacecraft, in which at least 0.000 001 % *of humanity can escape. To escape, people choose the star Tau Ceti which is* 12 ly *away at that time. They build engines that accelerate the spacecraft to a cruising speed of* $v = 0.75c$ *in very little time. Unfortunately, at half a distance to their destination, they observe both the Sun's and Tau Ceti's explosions. How long before the observation the explosions happened in an inertial coordinate frame of the spacecraft? And when in the coordinate system fixed with the stars? Presume that the Sun and Tau Ceti are static relative to each other.*

Karel wanted to escape in time, but it didn't work.

In the coordinate system associated with the stars, the ship reaches the halfway point at time $t_1 = \frac{d}{2v} = 8\,\mathrm{years}$ after leaving the Earth. At this time, the ship observed an explosion of both stars, whose light traveled $t_0 = \frac{d}{2c} = 6\,\mathrm{years}$ to the ship from both sides. Thus, in a system associated with stars, both explosions occurred $t_0 = 6\,\mathrm{years}$ before they were observed by the ship.

In a system associated with the ship, the situation is more complicated due to the influence of relativistic effects. In this system, the ship is static, while the Sun is moving away from it with the velocity v and Tau Ceti is approaching with the velocity v. Consider that the distance of the stars in the system associated with the ship is D. From the time the Sun exploded, the Sun traveled a distance vt_1 and light traveled ct_1 to the time the explosion was observed by the ship. The time t_1 indicates the time elapsed on the ship. Given that at the moment of observation, the ship is halfway between the stars, we can write the following equations

$$\frac{D}{2} = ct_1 + vt_1 = ct_2 - vt_2\,,$$

$$t_1 = \frac{D}{2(c+v)}\,, \qquad t_2 = \frac{D}{2(c-v)}\,,$$

where we have considered similarly for the second star the time t_2 and the fact that the light and the star have the same direction of motion. Thus, we see that the observer on the ship determines the times of the stellar explosions as not simultaneous. Moreover, even the distance of the stars observed in the ship's system is not the same as in the star system. These distances are linked by a relation for the so-called length contraction

$$D = d\sqrt{1 - \frac{v^2}{c^2}}\,.$$

Overall, for the times of the solar and Tau Ceti outbursts prior to their observation in the ship's system, we have

$$t_1 = \frac{d}{2(c+v)}\sqrt{1 - \frac{v^2}{c^2}} = \frac{d}{2c}\sqrt{\frac{c-v}{c+v}} = t_0\sqrt{\frac{c-v}{c+v}} \doteq 2.2\,\mathrm{years}\,,$$

$$t_2 = t_0\sqrt{\frac{c+v}{c-v}} = 15.9\,\mathrm{years}\,.$$

One could also come to this conclusion by making appropriate use of the formula for the Doppler shift by looking at a periodic signal with period t_0. Or by using full Lorentz transformation equation as the two points in spacetime (the explosion and the observation) are not at the same time or same point in space in neither of the two reference frames.

Let us note that the stars are moving towards each other at speeds in the order of ten kilometers per hour, so the assumption of mutual immobility is justified. A little less realistic is that at the end of its life, the Sun (and Tau Ceti) will go through a phase of the Asymptotic Giant Branch, which is accompanied by significant thermal pulses shedding large amounts of matter lasting several hundred thousand years. This matter will then form a planetary nebula around the Sun, revealing a core that will become a white dwarf. The Sun is too massive for a supernova explosion. Therefore, for life in the Solar system, a gradual increase in the radiant power of the Sun is significantly more dangerous as it will make life on Earth as we know it impossible in about a billion years.

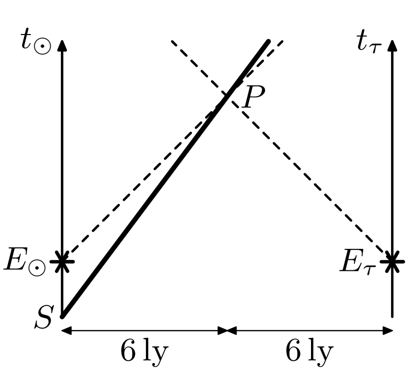

Fig. 1: Space time diagram in the stars reference frame, worldline of stars are vertical, of spaceship bold, of lightrays dashed. S denotes the start of the ship, E the explosions of stars and P the observation.

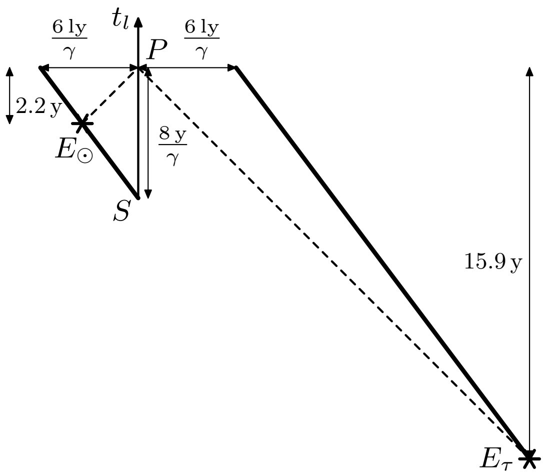

Fig. 2: Spacetime diagram in the spaceship reference frame after full Lorentz transformation as it would appear for the observer on board the ship.

Jozef Lipták
`liptak.j@fykos.org`

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