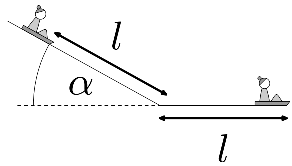
Problem III.3 ... bobsled

5 points; průměr 3,44; řešilo 89 studentů



Matěj and David are sliding on bobsleds down the hill. The hill with a slope of $\alpha = 29^\circ$ turns into the horizontal ground at the bottom of it. Both of them started from rest from the same height. Matěj's bobsled always travels the same distance l on an inclined plane as well as in a horizontal part. Since the bobsled digs deeper into the snow at higher loads, assume the coefficient of friction to be proportional to the normal force as $f(F) = kF$, where k is a positive constant. Determine how many times Matěj will travel farther from the bottom of the hill than David if David's mass (including the bobsled) is 12 % greater than Matěj's. Also, assume that bobsledders don't lose any energy at the bottom of the hill. *Matej likes to talk about bobsled.*

We can solve the problem using the law of conservation of energy. When the bobsledders are at the top of a hill with height h, they have potential energy E_p, which converses into kinetic energy E_k and work W_t done by friction. When the plane is reached, the kinetic energy E_k converses into more work done by friction W'_t. For the work done by going down the hill

$$W_t = F_t d = f F_\mathrm{N} d = k F_N^2 d\,,$$

where d is the distance traveled and F_N is the normal force, which in this case equals $F_N = mg\cos\alpha$.

Let us first consider the motion of Matej, who is known to travel the same distance on the hill and the plain. Let us denote it by l. From the geometry of the hill, it is clear that its height will be $h = l\sin\alpha$. Let us write the two equations mentioned above

$$E_{pM} = W_{tM} + E_{kM}\,,$$
$$E_{kM} = W'_{tM}\,.$$

which, when substituted, have the form

$$mgl\sin\alpha = k(mg\cos\alpha)^2 l + E_{kM}\,,$$
$$E_{kM} = k(mg)^2 l\,.$$

Next, we plug the second equation into the first one

$$mgl\sin\alpha = k(mg\cos\alpha)^2 l + k(mg)^2 l\,,$$

where, after adjustments, we express the coefficient k as

$$k = \frac{1}{mg}\frac{\sin\alpha}{1+\cos^2\alpha}\,.$$

We will do the same for David, whose mass is $m_D = 1.12\,m$, and we denote the path he travels on the plane as l_D. We get

$$1.12\,mgl\sin\alpha = k(1.12\,mg\cos\alpha)^2 l + E_{kD}\,,$$
$$E_{kD} = k(1.12\,mg)^2 l_D\,.$$

We again express the kinetic energy plus the coefficient k

$$1.12\,mgl\sin\alpha = \frac{1}{mg}\frac{\sin\alpha}{1+\cos^2\alpha}(1.12\,mg\cos\alpha)^2 l + \frac{1}{mg}\frac{\sin\alpha}{1+\cos^2\alpha}(1.12\,mg)^2 l_D\,.$$

and finally find the ratio $\frac{l}{l_D}$ as

$$\frac{l}{l_D} = \frac{1.12}{1-0.12\,\cos^2\alpha}\,.$$

Just substitute in the angle $\alpha = 29^\circ$ and you have the result

$$\frac{l}{l_D} \doteq 1.23\,.$$

Patrik Kašpárek
`patrik.kasparek@fykos.org`

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