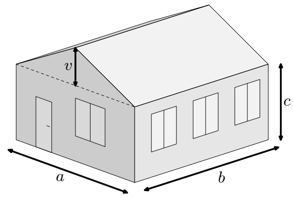
Problem III.2 ... heating in the cottage
3 points; průměr 2,68; řešilo 130 studentů



Danka arrived at her cottage in the middle of winter with only $T_1 = 12\,^\circ\mathrm{C}$ indoors. So she lit a fire in the fireplace by using wood of heating value $Q_0 = 14.23\,\mathrm{MJ{\cdot}kg^{-1}}$. How much wood does she need to burn to heat the air inside to $T_2 = 20\,^\circ\mathrm{C}$? The cottage is in the shape of a rectangular cuboid with dimensions $a = 6\,\mathrm{m}$, $b = 8\,\mathrm{m}$ and $c = 3\,\mathrm{m}$. A roof is in the shape of an irregular recumbent triangular prism with a height of $v = 1.5\,\mathrm{m}$, the upper edge of which is the axis of the cottage layout. The air occupies $87\,\%$ of the volume of the cottage and its specific heat capacity is $c_\mathrm{a} = 1\,007\mathrm{J{\cdot}kg^{-1}{\cdot}K^{-1}}$. Does the result match the expectation? Discuss the simplicity of the model used. *Danka gets cold at the cottage.*

To heat the air in the cottage, Danka has to get energy by burning wood. If she burns wood of mass M, which will release a heat energy

$$Q = MH\,.$$

If all this heat is transferred to the air, it satisfies

$$Q = mc_\mathrm{a}(T_2 - T_1)\,,$$

where m is the mass of the air in the cottage. What we can easily calculate as the product of the volume and the density of the air. The total volume of the cottage V_0 is the sum of the cube volume defined by the cottage's walls and the triangular prism being the cottage's roof. The prism's volume equals $\frac{1}{2}avb$. Consequently, the volume of the cottage is

$$V_0 = abc + \frac{1}{2}avb = ab(c + \frac{1}{2}v)\,.$$

Since we know that the air occupies only $\eta = 0.87$ of the volume of the whole cottage, we can express this as

$$Q = \eta ab(c + \frac{1}{2}v)\rho_\mathrm{a}c_\mathrm{a}(T_2 - T_1)\,.$$

Finally, we substitute back the initial equations from the beginning, express the required mass of wood, and get

$$M = \frac{\eta\rho_\mathrm{a}c_\mathrm{a}}{H}ab(c + \frac{1}{2}v)(T_2 - T_1)\,.$$

After substituting the given values, we get that Danka only needs to burn $M \approx 0.11\,\mathrm{kg}$ of wood.

When we think about it, we realize that in real life, it does not work like that. It is certainly not enough to burn just a hundred grams of wood to warm the air inside the cottage by a few degrees. From our experiences, we would instead expect at least a few kilograms. Our simple model is wrong. It assumes that all the heat generated by burning the wood is used to heat the air, and only the air.

As a matter of fact, several other factors play a role in heating the cottage. The most noticeable of these is that the efficiency of the heat transfer is far from one hundred percent. A large amount of warm air escapes through the chimney and out of the cottage. The chimney's walls will heat up and transfer heat to the surrounding air in the cottage. We should keep in mind that the specific heat capacity of a concrete wall has similar order of magnitude as the specific heat capacity of the air [1]; however, the wall has several orders of magnitude more. Therefore, the total heat required to heat the wall is much greater than that needed for heating the air. Similarly, the objects inside the cottage will consume the heat. If the cottage is furnished and the furniture is made of wood, plastic, or cloth, which will absorb much more heat than the air. Although the problem statement mentioned heating just the air, we cannot neglect the heat exchange between the air in the cottage and the objects inside. As long as the objects are colder than the air, heat will flow from the air into these objects, thereby cooling the air. In order to achieve a stable air temperature in the room, all the objects must reach temperature. Another factor that we did not take into account is the thermal insulation of the cottage. We have assumed that the cottage is thermally isolated. However, in reality, inside a cottage takes place a significant heat exchange with the surroundings takes place. The extent of the heat exchange depends on the cottage's material, the quality of the walls' insulation, and how well the windows and doors are sealed.

If we consider all these factors, it would be possible to calculate the actual mass of burnt wood needed to heat the air in the cottage. Then the quotient obtained by dividing the simplified model mentioned above and the actual value would describe the efficiency-like variable of the energy transferred from the wood to the air. As we can see by the unrealistically small mass obtained by the calculation, this efficiency is very low.

Daniela Dupkalová
`daniela@fykos.org`

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[1] `https://www.designingbuildings.co.uk/wiki/Specific_heat_capacity`