Problem II.2 ... uncomfortable bus   3 points; (chybí statistiky)

Jarda wanted to watch a lecture on his laptop on the bus, so he put the laptop on a flip-up shelf of the seat in front of him. The shelf has a depth $h = = 18$ cm and is perpendicular to the vertical backrest. Jarda's laptop, which is $l = 25$ cm wide, consists of a base weighing $M = 1\,200$ g, and a screen weighing $m = 650$ g. Let us assume that both parts are of homogeneous density. What is the largest angle the laptop can open up without falling off the shelf?
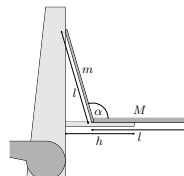
Jarda is a workaholic.

To prevent Jarda's laptop from falling off the shelf, its center of gravity must be above the shelf. The most extreme situation is when the center of gravity lies above the end of the shelf, in the distance $h = 18$ cm from the seat.

As we want the angle to be as large as possible, the laptop's screen should be in touch with the seat. Assume that between the plane of the screen and the plane of the shelf is an obtuse angle α (keep in mind that $\cos(\alpha) = -\cos(\pi - \alpha)$ holds, so the cosine of a supplementary angle has an opposite sign). Then horizontal distance from the seat to the centre of laptop's gravity is $x_m = -(l\cos\alpha)/2$, where $l = 25$ cm is a dimension of the laptop. The centre of gravity of the laptop's base is in distance $x_M = -l\cos\alpha + l/2$ from the seat. Hence, the center of gravity is in the distance

$$x = \frac{x_m m + x_M M}{m + M} = \frac{-ml\cos\alpha - 2Ml\cos\alpha + Ml}{2\,(M + m)} = h$$

from the seat, we get the last equality from the abovementioned idea.

Now we can isolate α as

$$\alpha = \arccos\left(\frac{Ml - 2\,(M + m)\,h}{ml + 2Ml}\right) = 120\,^\circ\,.$$

The largest angle the laptop can open up to without falling off the shelf is only $120\,^\circ$, so watching lectures on a bus is not very comfortable because the shelf is quite low, and Jarda can't achieve a perpendicular view of the screen.

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